MATH 322 - SEC 001, SPRING 2013. HOMEWORK 7

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Due : Friday, March 22

Please show all your work and/or justify your answers for full credit.

Problem 1: (*Textbook problem 3.3.2*) For the following functions, sketch the Fourier sine series of $f(x)$ and determine its Fourier coefficients.

(a) $f(x) = \cos\left(\frac{\pi x}{L}\right)$

(b) $f(x) = \begin{cases} 1 & x < L/6 \\ 3 & L/6 < x < L/2 \\ 0 & x > L/2 \end{cases}$

Problem 2: (*Textbook problem 3.3.5*) For the following functions, sketch the Fourier cosine series of $f(x)$ and determine its Fourier coefficients

(a) $f(x) \begin{cases} 1 & x < L/6 \\ 3 & L/6 < x < L/2 \\ 0 & x > L/2 \end{cases}$

(b) $f(x) = \begin{cases} 0, & x < L/2 \\ x, & x > L/2 \end{cases}$

Problem 3: (*Textbook problem 3.3.9*) What is the sum of the Fourier sine series of $f(x)$ and the Fourier cosine series of $f(x)$? What is the sum of the even and odd extension of $f(x)$?

Problem 4: (*Textbook problem 3.3.18*) **For full credit, explain your conclusions**. For continuous functions,

(a) Under what conditions does $f(x)$ equal its Fourier series for all x, $-L \leq x \leq L$?

(b) Under what conditions does $f(x)$ equal its Fourier sine series for all x, $0 \leq x \leq L$?

(c) Under what conditions does $f(x)$ equal its Fourier cosine series for all x, $0 \leq x \leq L$?

Problem 5: *Textbook problem 3.4.6*. There are some things wrong in the following demonstration. Find the mistakes and correct them.

In this exercise we attempt to obtain the Fourier cosine coefficients of e^x:

$$e^x = A_0 + \sum_{n=1}^{\infty} A_n \cos\left(\frac{n\pi x}{L}\right) \tag{0.0.1}$$

Differentiating yields

$$e^x = -\sum_{n=1}^{\infty} \frac{n\pi}{L} A_n \sin\left(\frac{n\pi x}{L}\right),$$

the Fourier sine series of e^x. Differentiating again yields

$$e^x = -\sum_{n=1}^{\infty} \left(\frac{n\pi}{L}\right)^2 A_n \cos\left(\frac{n\pi x}{L}\right). \tag{0.0.2}$$

Since equations (0.0.1) and (0.0.2) give the Fourier cosine series of e^x, they must be identical. Thus,

$$\left.\begin{array}{l} A_0 = 0 \\ A_n = 0 \end{array}\right\} \text{obviously wrong!.}$$

By correcting the mistakes, you should be able to obtain A_0 and A_n without using the typical technique, that is, $A_n = 2/L \int_0^L e^x \cos(n\pi x/L)dx$.

Problem 6: *Textbook problem 4.2.1*

(a) Using equation (4.2.7) in the textbook, compute the sagged equilibrium position $u_E(x)$ if $Q(x,t) = -g$. The boundary conditions are $u(0) = 0$ and $u(L) = 0$.

(b) Show that $v(x,t) = u(x,t) - u_E(x)$ satisfies the equation 4.2.9 in the textbook