MATH 322 - SEC 001, SPRING 2013. HOMEWORK 6

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Due : Friday, March 15

Please show all your work and/or justify your answers for full credit.

Problem 1:

(a) Set $x = \frac{\pi}{2}$ in the Fourier series of $f(x) = x, -\pi < x < \pi$, to obtain the formula

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

(b) Set $x = \frac{\pi}{4}$ in the series of part (a) to obtain

$$\frac{\pi}{4} = \sqrt{2}\left(1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \dots\right) - \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\right)$$

(c) Conclude from part (b) that

$$\frac{\pi}{2\sqrt{2}} = 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \frac{1}{13} - \frac{1}{15} + \dots$$

(d) If we set $x = \pi$ in the series in part (a), we find that the series sums to zero. Why doesn't it contradict $f(x) = x$?

Problem 2: From homework 5 (problem 1) we know that

$$x^2 \sim \frac{\pi^2}{3} - 4\cos x + \cos 2x - \frac{4}{9}\cos 3x + \dots + (-1)^m \frac{4}{m^2}\cos(mx) + \dots$$

for $-\pi \le x \le \pi$.

(a) Setting $x = 0$, find the sum

$$1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots = \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{m^2}$$

(b) What is

$$\sum_{n=1}^{\infty} \frac{1}{m^2}$$

Problem 3: Let $f(x), -L < x < L$ be a piecewise smooth function with Fourier series

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right].$$

Show that $a_n = O(1/n)$ and $b_n = O(1/n)$ are both of order $1/n$ when $n \to \infty$. That it, show that na_n and nb_n are bounded as $n \to \infty$.

Problem 4: Among the series for x, x^2 and $x^3 - L^2 x$, $-L < x < L$, which can be differentiated term by term obtaining the derivative of the original series?

Problem 5: Let $f(x) = x(\pi - x), 0 \le x \le \pi$.

(a) Compute the Fourier sine series of f.

(b) Compute the Fourier cosine series of f.

(c) Find the mean square error incurred by using N terms of each series and find asymptotic estimates when $N \to \infty$

(d) Which series gives a better mean square approximation of f?