MATH 322 - SEC 001, SPRING 2013. HOMEWORK 5

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Due : Friday, March 8

Please show all your work and/or justify your answers for full credit.

Problem 1: Compute the Fourier series of the indicated functions

(a) $f(x) = x^2$, $-L \leq x < L$
(b) $f(x) = x^3$, $-L \leq x < L$
(c) $f(x) = |x|^3$, $-L \leq x < L$
(d) $f(x) = \sin^2(2x)$, $-L \leq x < L$

Problem 2: Prove the following fact about even and odd functions

(a) The product of two even functions is even
(b) The product of two odd functions is even
(c) The product of an even function and an odd function is odd
(d) Which of the statements (a),(b), (c) remain true if the world "product" is replaced by "sum"?

Problem 3: Le f be an arbitrary function. Show that there is an odd function f_1 and an even function f_2 such that $f = f_1 + f_2$.

Problem 4: Which if the following functions are even, odd, or neither?

(a) $f(x) = \cos(3x)$
(b) $f(x) = x^3 - 3x$
(c) $f(x) = \sin x - 3x^5$
(d) $f(x) = x^2 - \cos(x)$
(e) $f(x) = x^3 - x^2$
(f) $f(x) = |x| \sin x$

Problem 5: Let $f(x)$, $-L \leq x < L$, be an odd function that satisfies the symmetry condition

$$f(L - x) = f(x).$$

Show that the sine and cosine coefficients in the Fourier series $A_0 + \sum_{n=1}^{\infty} \left(A_n \cos\left(\frac{n\pi x}{L}\right) + B_n \sin\left(\frac{n\pi x}{L}\right)\right)$ satsify

$$A_n = 0 \text{ for all n, and } B_n = 0 \text{ for all even n}$$

Problem 6: For each of the functions below, state wether or not it is periodic and find the smallest period.

(a) $f(x) = \sin(\pi x)$
(b) $f(x) = \sin(2x) + \sin(3x)$
(c) $f(x) = \sin(x) + \sin(\pi x)$
(d) $f(x) = x - [x]$, where $[x]$ = integer part of x
(e) $f(x) = \sum_{n=1}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(f) $f(x) = \sin(x^2)$