MATH 322 - SEC 001, SPRING 2013. HOMEWORK 11

INSTRUCTOR: GERARDO HERNÁNDEZ

Due : Friday, May 3rd

Please show all your work and/or justify your answers for full credit.

Problem 1: (*Textbook problem 5.8.6*) Consider (with $h > 0$)

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2}{\partial x^2} \\ \begin{cases} \frac{\partial u}{\partial x}(0,t) - h\ u(0,t) = 0 \\ \frac{\partial u}{\partial x}(L,t) = 0 \end{cases} \\ \begin{cases} u(x,0) = f(x) \\ \frac{\partial u}{\partial t}(x,0) = g(x). \end{cases} \end{cases}$$

(a) Show that there are an infinite number of different frequencies of oscillation.
(b) Estimate the large frequencies of oscillation
(c) Solve the initial value problem

Problem 2: (*Textbook problem 5.8.8*) Consider the boundary value problem

$$\begin{cases} \frac{d^2\phi}{dx^2} + \lambda\phi = 0 \\ \phi(0) - \frac{d\phi}{dx}(0) = 0 \\ \phi(1) + \frac{d\phi}{dx}(1) = 0. \end{cases}$$

(a) Using the Rayleigh quotient, show that $\lambda \geq 0$. Why is $\lambda > 0$?
(b) Prove that eigenfunctions corresponding to different eigenvalues are orthogonal
(c) Show that

$$\tan\sqrt{\lambda} = \frac{2\sqrt{\lambda}}{\lambda - 1}.$$

Determine the eigenvalues graphically. Estimate the large eigenvalues (using the graph).

(d) Solve

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

with

$$\begin{cases} u(0,t) - \frac{\partial u}{\partial x}(0,t) = 0 \\ u(1,t) + \frac{\partial u}{\partial x}(1,t) = 0 \\ u(x,0) = f(x). \end{cases}$$

You may call the relevant eigenfunctions $\phi_n(x)$ and assume that they are known.

Problem 3: (*Textbook problem 5.9.1*) Estimate (to leading order) the large eigenvalues and corresponding eigenfunctions for

$$\frac{d}{dx}\left(p(x)\frac{d\phi}{dx}\right) + \left[\lambda\sigma(x) + q(x)\right]\phi = 0$$

if the boundary conditions are

(a)
$$\frac{d\phi}{dx}(0) = 0 \text{ and } \frac{d\phi}{dx}(L) = 0.$$

(b)
$$\phi(0) = 0 \text{ and } \frac{d\phi}{dx}(L) = 0.$$

(c)
$$\phi(0) = 0 \text{ and } \frac{d\phi}{dx}(L) + h\phi(L) = 0.$$