

MATH 320 - SEC 001, SPRING 2012. PRACTICE EXAM 3

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1. Consider the system of equations

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} -3 & 5 \\ -5 & 3 \end{bmatrix} \mathbf{x}$$

- (a) Find the general solution
- (b) Write the solution in terms of real functions only

2. Given

$$y^{(5)} - 8y^{(4)} + 16y^{(3)} + y'' - 8y' + 16y = x^2 \exp(4x)$$

- (a) Find the homogeneous solution given that the characteristic equation is  $r^5 - 8r^4 + 16r^3 + r^2 - 8r + 16 = (r^3 + 1)(r^2 - 8r + 16)$ .
- (b) Write down the form of the particular solution. **Do not solve for the coefficients.**

3. Find a general solution to

$$y'' + y' + \frac{1}{4}y = \frac{1}{5}t^{-2} \exp(-t/2), \quad t > 0.$$

4. Consider the system of equations

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} 2 & 0 & 0 \\ -7 & 9 & 7 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x}$$

- (a) Find the homogeneous solution.
- (b) Find the Fundamental Matrix and its inverse.
- (c) Use the Fundamental Matrix and its inverse to find the solution that satisfies the following initial condition:

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

5. Let  $A$  be the  $3 \times 3$  matrix given by

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

- (a) Find the eigenvalues and corresponding complete set of eigenvectors
- (b) Diagonalize the matrix  $A$  and use it to compute the power  $A^{500}$  of the matrix  $A$ .

6. Compute the matrix exponential  $e^{At}$ , where

$$A = \begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}$$