MATH 320 - SEC 001, SPRING 2012. PRACTICE EXAM 2

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1. Given that two vectors $\mathbf{u}$ and $\mathbf{v}$ are linearly independent, are $\mathbf{u} - \mathbf{v}$ and $\mathbf{v}$ linearly dependent or linearly independent? Prove your answer.

2.

(a) For what vectors $\mathbf{b}$ does $\mathbf{Ax} = \mathbf{b}$ have a solution, with $\mathbf{A}$ given by

$$\mathbf{A} = \begin{bmatrix} 6 & 3 & 3 \\ 2 & 5 & -1 \\ -4 & -8 & 1 \end{bmatrix}$$

(b) Find a basis for the vector space spanned by the columns of $\mathbf{A}$.

(c) Find all possible solutions for $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -3/2 \end{bmatrix}$.

3. Find the determinant of the following matrix using elementary row operations:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 & 5 \\ -1 & 2 & 3 & 4 \\ 1 & 3 & 1 & -2 \\ -1 & -3 & 0 & -4 \end{bmatrix}$$

4. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$. Find a basis for $\mathbb{R}^4$ **containing** the vectors v_1 and v_2.

5. Let $\mathbf{A}$ and $\mathbf{B}$ be $n \times n$ matrices. Show that $\mathbf{AB}$ is invertible if and only if both $\mathbf{A}$ and $\mathbf{B}$ are invertible.