

Section 3.1 #10

$$x + 3y + 2z = 2$$

$$2x + 7y + 7z = -1$$

$$2x + 5y + 2z = 7$$

Augmented coefficient matrix:

$$\begin{bmatrix} 1 & 3 & 2 & 2 \\ 2 & 7 & 7 & -1 \\ 2 & 5 & 2 & 7 \end{bmatrix} \xrightarrow{(-2)R_1 + R_2} \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 2 & 5 & 2 & 7 \end{bmatrix}$$

$$\xrightarrow{(-2)R_1 + R_3} \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & -1 & -2 & 3 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

\Rightarrow consistent: no free variables

Back substitution:

$$x_3 = -2, \quad x_2 + 3x_3 = -5 \Rightarrow x_2 = -3x_3 - 5 = -3(-2) - 5 = 1$$

$$x_1 + 3x_2 + 2x_3 = 2$$

$$x_1 + 3 \cdot 1 + 2 \cdot (-2) = 2 \quad x_1 = 2 + 1 = 3$$

$$\Rightarrow \begin{aligned} x &= 3 \\ y &= 1 \\ z &= -2 \end{aligned}$$