MATH 319 - SEC 003, SPRING 2014. HOMEWORK 9

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Due : Friday, April 11.

Please show all your work and/or justify your answers.

Section 5.2 Problems 19

(a) By making the change of variables $x - 1 = t$ and assuming y has a Taylor series in powers of t, find two series solutions of

$$y'' + (x - 1)^2 y' + (x^2 - 1)y = 0$$

in powers of $x - 1$

(b) Show that you obtain the same result by assuming y has a Taylor series in powers of $x - 1$ and also expressing the coefficient $x^2 - 1$ in powers of $x - 1$.

Section 5.2 Problems 20 Show directly, using the ratio test, that the two series solutions of Airy's equation about $x = 0$ converge for all x; see Eq. (20) of the text.

Section 5.2 Problems 21 The Hermite Equation. The equation

$$y'' - 2xy' + \lambda y = 0, -\infty < x < \infty$$

where λ is a constant, is known ad the Hermite equation. It is an important equation in mathematical physics.

(a) Find the first four terms in each of two solutions about $x = 0$ and show that they form a fundamental set of solution

(b) Observe that if λ is a non negative even integer, then one or the other of the series solutions terminates and becomes a polynomial. Find the polynomial solutions for $\lambda = 0, 2, 4, 6, 8$ and 10. Note that each polynomial is determined uo to a multiplicative constant.

(c) The Hermite polynomial $H_n(x)$ is defined as the polynomial solution of the Hermite equation with $\lambda = 2n$ for which the coefficient of x^n is 2^n. Find $H_0(x), \ldots, H_5(x)$.