MATH 319 - SEC 003, SPRING 2014. HOMEWORK 8

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Due : Wednesday, April 2nd.

Please show all your work and/or justify your answers.

Section 4.1 Problems 11 and 16 Verify that the given functions are solutions of the differential equation, and determine their Wronskian.

11. $y''' + y' = 0$; $1, \cos(t), \sin(t)$

16. $x^3y''' + x^2y'' - 2xy' + 2y = 0$; $x, x^2, 1/x$.

Problem Consider the Hermite differential equation

$$y'' - 2xy' + y = 0, \; y(0) = y_0, y'(0) = y_0'.$$

Find the recursive relation for the power series solution. Compute the first 7 terms.

Section 5.1 Problems 1,4, and 8 Determine the radius of convergence of the given power series

- $\Sigma_{n=0}^{\infty}(x-3)^n$
- $\Sigma_{n=0}^{\infty}2^n x^n$
- $\Sigma_{n=1}^{\infty}\frac{n!x^n}{n^n}$

Section 5.1 Problems 9, 12 Determine the Taylor series about the point x_0 for the given function. Also determine the radius of convergence of the series.

- $\sin x$, $x_0 = 0$
- x^2; $x_0 = -1$.

Section 5.1 Problems 21, 26 Rewrite the given expression as a sum whose generic term involves x^n

- $\Sigma_{n=2}^{\infty}n(n-1)a_n x^{n-2}$
- $\Sigma_{n=1}^{\infty}na_n x^{n-1} + x\Sigma_{n=0}^{\infty}a_n x^n$

Section 5.2 Problems 2,4,5,8. In each of the following problems,

(a) Seek power series solutions of the given DE about the given point x_0; find the recurrence relation
(b) Find the first four terms in each of the two solutions y_1, y_2
(c) By evaluating the Wronskian, show that y_1 and y_2 form a fundamental set of solutions.
(d) If possible, find the general term in each solution

- $y'' - xy' - y = 0$, $x_0 = 0$
- $y'' + k^2x^2y = 0$, $x_0 = 0$, k a constant
- $(1-x)y'' + y = 0$, $x_0 = 0$
- $xy'' + y' + xy = 0$, $x_0 = 1$