MATH 319 - SEC 003, SPRING 2014. HOMEWORK 11

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Due : Friday, April 25.

Please show all your work and/or justify your answers.

Section 5.5 Problems 5,6,7 In each of the problems 5,6,7

(a) Show that the given differential equation has a regular singular point at $x = 0$

(b) Determine the indicial equation, the recurrence relation, and the roots of the indicial equation

(c) Find the series solution ($x > 0$) corresponding to the larger root

(d) If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

5. $3x^2y'' + 2xy' + x^2y = 0$

6. $x^2y'' + xy' + (x - 2)y = 0$

7. $xy'' + (1 - x)y' - y = 0$

Section 5.5 Problem 14 The Bessel equation of order zero is

$$x^2y'' + xy' + x^2y = 0.$$

(a) Shows that $x = 0$ is a regular singular point.

(b) show that the roots of the indicial equation are $r_1 = r_2 = 0$.

(c) Show that one solution for $x > 0$ is

$$J_0(x) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

(d) Show that the series of $J_0(x)$ converges for all x. The function J_0 is known as the Bessel function of the first kind of order zero.

Section 5.5 Problem 15 Referring o Problem 14, use the method of reduction of order to show that the second solution of the Bessel equation of order zero contains a logarithmic term.
Hint: If $y_2(x) = J_0(x)v(x)$, then

$$y_2(x) = J_0(x) \int \frac{dx}{x[J_0(x)]^2}.$$

Find the first term in the series expansion of $\frac{1}{x[J_0(x)]^2}$.