Fundamentos de Matemáticas para Materiales

Tarea 3:

Problema 1: Determina si cada una de las siguientes series convergen o divergen. Usa cualquier prueba que consideres apropiada.

(a) $\sum_{n=1}^{\infty} (n+3)^{-3/2}$

R: $a_n = (n+3)^{-3/2} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{(n+4)^{-3/2}}{(n+3)^{-3/2}} = \left(\frac{n+3}{n+4}\right)^{+3/2} = \left(1 - \frac{1}{n+4}\right)^{3/2}$

$\sim 1 + \frac{3}{2}\left(-\frac{1}{n+4}\right) = 1 - \frac{3/2}{n+4} \underset{n\to\infty}{\sim} 1 - \frac{3/2}{n} = 1 - \frac{r'}{n}, \; r' = 3/2 >$

$\Rightarrow$ converge.

(b) $\sum_{n=1}^{\infty} e^{1/n}$ $\quad a_n = e^{1/n}$

R: $\frac{a_{n+1}}{a_n} = \frac{e^{\frac{1}{n+1}}}{e^{1/n}} = e^{1/n+1 - 1/n} = e^{-\frac{1}{n(n+1)}}$

$\sim 1 - \frac{1}{n(n+1)} \sim 1 - \frac{1}{n^2} \Rightarrow$ diverge.

(c) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ $\quad a_n = \frac{n!}{n^n}$

R: $\frac{a_{n+1}}{a_n} = \frac{(n+1)!/(n+1)^{n+1}}{n!/n^n} = \frac{(n+1)\,n^n}{(n+1)^{n+1}} = \frac{n^n}{(n+1)^n} = \left(1 - \frac{1}{n+1}\right)^n \sim e^{-1} < 1$

$\Rightarrow$ converge.

(d) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right)$ R: $1 + \frac{1}{n^2} > 1 \quad \forall n \in \mathbb{N} \Rightarrow$ diverge

(e) $\sum_{n=1}^{\infty} \frac{n^2+2n-1}{n^4+3}$ R: $a_n = \frac{n^2+2n-1}{n^4+3} \sim \frac{1}{n^2} \Rightarrow$ converge

(f) $\sum_{n=1}^{\infty} n^{-1/00}$ $\quad a_n = n^{-100}$ $\quad r' = 100 > 1 \Rightarrow$ converge.

R: $\frac{a_{n+1}}{a_n} = \left(1 - \frac{1}{n+1}\right)^{100} \sim 1 - \frac{100}{n+1} \sim 1 - \frac{r'}{n+1}$

Problema 2: Determina la convergencia absoluta, condicional, o divergencia de las siguientes series:

(a) $\sum_{n=1}^{\infty} (-1)^n \ln\left(1+\frac{1}{\sqrt{n}}\right)$

R: $\ln\left(1+\frac{1}{\sqrt{n}}\right) \xrightarrow[n\to\infty]{} 0$ y $\sum (-1)^n$ tiene sumas parciales acotadas.

$\Rightarrow$ Por la prueba de Dirichlet, la suma converge

(b) $\sum_{n=1}^{\infty} \frac{(-1)}{2^n}$

R: La suma converge absolutamente

$$\frac{\frac{1}{2^{n+1}}}{\frac{1}{2^n}} = \frac{1}{2} < 1$$

Problema 3: Determina el intervalo de convergencia en x de las siguientes series de potencia.

(a) $\sum_{n=1}^{\infty} n!\, x^n$.

R: $\frac{(n+1)!\, x^{n+1}}{n!\, x^n} = (n+1)x \Rightarrow$ el radio de convergencia es cero.

(b) $\sum \left[\ln(1+n^2)\right] x^n$

$$\frac{\ln(1+(n+1)^2)}{\ln(1+n^2)} \approx \frac{2\ln(n+1)}{2\ln(n)} \sim \frac{\ln\left(\frac{1}{n+1}\right)}{\ln\left(\frac{1}{n}\right)} \sim \frac{\frac{1}{n+1} - \frac{1}{(n+1)^2}}{\frac{1}{n} - \frac{1}{n^2}}$$

$$= \frac{n^2(n+1-1)}{(n+1)^2(n-1)} \sim 1 \qquad \Rightarrow \text{radio} = 1.$$