Hw3.
2014

Tarea 3:

Problema 3

Encuentra $\iint_S \vec{F} \cdot d\vec{S}$ para

$\vec{F}(x,y,z) = (xy, 4x^2, yz)$, S la sup. $z = xe^y$
$0 \le x \le 1$
$0 \le y \le 1.$

$$\iint_S \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^1 \vec{F} \cdot \vec{r}_x \times \vec{r}_y$$

$\vec{r}(x,y) = (x, y, xe^y)$

$\vec{r}_x = (1, 0, e^y)$

$\vec{r}_y = (0, 1, xe^y)$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} i & j & k \\ 1 & 0 & e^y \\ 0 & 1 & xe^y \end{vmatrix}$$

$$= i(-e^y) - j \cdot (xe^y) + k = (-e^y, -xe^y, 1)$$

$$\iint_S \vec{F} \cdot d\vec{S} = \int_0^1 \int_0^1 -x\cancel{y}e^y - 4x^3 e^y + \cancel{yxe^y} \, dx\, dy$$

$$= -x^4\Big|_0^1 e^y\Big|_0^1 = -1 \cdot (e-1) = 1-e$$

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Problema 4 Encuentra la masa del embudo delgado con la forma de un cono $z = \sqrt{x^2+y^2}$, $1 \le z \le 4$, si la densidad es $\rho = 10 - z$.

$1 \le x^2+y^2 \le 16$

$x = r\cos\theta$
$y = r\,sen\theta$
$z = r$

$0 \le \theta \le 2\pi$
$1 \le z \le 4$

~~Masa = $\int_0^{2\pi}$~~ $\vec{r} = (r\cos\theta, r\,sen\theta, r)$

$\vec{r}_r = (\cos\theta, sen\theta, 1)$, $\vec{r}_\theta = (-r\,sen\theta, r\cos\theta, 0)$

$$\vec{r}_r \times \vec{r}_\theta = \begin{vmatrix} i & j & k \\ \cos\theta & \operatorname{sen}\theta & 1 \\ -r\operatorname{sen}\theta & r\cos\theta & 0 \end{vmatrix} = i\,(-r\cos\theta) - j\,(r\operatorname{sen}\theta) + k\,r$$

$$= (-r\cos\theta, -r\operatorname{sen}\theta, r)$$

$$|\vec{r}_r \times \vec{r}_\theta| = \sqrt{r^2+r^2} = \sqrt{2}\, r$$

$$\text{Masa} = \int_0^{2\pi}\int_1^4 \sqrt{2}\, r\,(10-r)\,dr\,d\theta = 2\pi\sqrt{2}\left(10\frac{r^2}{2} - \frac{r^3}{3}\right)\Big|_1^4$$

$$= 2\sqrt{2}\,\pi\left(5\,(16-1) - \frac{64-1}{3}\right) = \frac{2\sqrt{2}\,\pi}{3}(225-63)$$

$$= \frac{2\sqrt{2}\,\pi}{3}\times 162 = 108\sqrt{2}\,\pi.$$

Problema 5: La temperatura en un punto en una bola con conductividad k es inversamente proporcional a su distancia dentro de la bola. Encuentra la razón de flujo de calor a través de la esfera S de radio a con centro en la bola.

centro = origen.

$$\vec{F} = -k\nabla u, \quad u = \frac{\alpha}{|x|}$$

$$u = \frac{\alpha}{\sqrt{x^2+y^2+z^2}} = \alpha\,(x^2+y^2+z^2)^{-1/2} \qquad u_x = (-\tfrac{1}{2})(\quad)^{-3/2}\,2x = \frac{-\cancel{2}x}{|\vec{x}|^3}$$

$$\vec{F} = +\frac{\cancel{2}\alpha k}{(x^2+y^2+z^2)^{3/2}}(x,y,z)$$

$x = a\cos\theta\cos\phi$ $\qquad 0 \le \theta \le 2\pi$

$y = a\operatorname{sen}\theta\,\cancel{\cos}\phi$ $\qquad -\pi/2 \le \phi \le \pi/2$

$z = a\operatorname{sen}\phi$

$$\vec{r}_\theta \times \vec{r}_\phi = a^2\cos\phi\,(\cos\theta\cos\phi, \operatorname{sen}\theta\cos\phi, \operatorname{sen}\phi)$$

$$\vec{F}\cdot\vec{r}_\theta\times\vec{r}_\phi = +\frac{2\alpha k}{|\vec{x}|^3}\,\vec{x}\cdot\left(a\cos\phi\,\vec{x}\right)$$

$$= +\frac{2\alpha k}{|\vec{x}|^3}\,a\cos\phi\,|\vec{x}|^2 = -\frac{2\alpha K\,a\cos\phi}{a}$$

$$= +2\alpha k\cos\phi$$

Flujo: $$\iint_S \vec{F}\cdot d\vec{S} = \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} +2\alpha k\cos\phi\,d\phi\,d\theta$$

$$= +2\pi\alpha k\,\operatorname{sen}\phi\Big|_{-\pi/2}^{\pi/2} = +4\pi\alpha K.$$