**Name:**___________________________________.

# MATH 115 - SEC 011, WINTER 2011. QUIZ 4
# TIME LIMIT: 30 MINUTES

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**Good luck!**

**Problem 1**

(1) State the *definition* of the derivative of a function $s(t)$ at $t = a$.

The derivative of a function $s(t)$ at $t = a$ is the **slope** of the tangent line to the graph of $s(t)$ at $t = a$, and is obtained by taking the limit when $h \to 0$ of the average rate of change:

$$s'(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

(2) Suppose that the position of a track star at UM (in meters) $t$ seconds after the start of the race is given by $s(t) = t^{\ln(t)}$. Write out the definition of $s'(1)$, and simplify as much as possible.

$$s'(1) = \lim_{h \to 0} \frac{(1+h)^{ln(1+h)} - 1}{h}$$

(3) Using your work from the previous part, estimate the value of $s'(1)$. Show your work!

It is enough to take an small enough $h$ to obtain an approximation. Take for example $h = 0.00001$, so we get

$$s'(1) \approx \frac{(1.00001)^{ln(1.00001)} - 1}{0.00001} = 0.00001$$

So it seems to approach zero.

*Date*: February 2nd, 2011.

**Problem 2**

A company's revenue from car sales, $C$ (in thousand of dollars), is a function of the advertising expenditure, $a$, in thousand of dollars, so $C = f(a)$

(a) What does the company hope is true about the sign of $f'$?

They expect the the revenue $C$ to increase as they expend more on advertising. So, they expect the sign of $f'$ to be positive.

(b) What does the statement $f'(100) = 2$ mean in practical terms? **Include units**.

In practical terms, it means

$$f(101) \approx f(100) + \$2000.$$

This says that the revenue from car sales when expending \$101,000 on advertising is approximately the revenue from car sales when the company expends \$100,000 on advertising, plus \$2,000.

**Problem 3**

Let $f(t)$ be the number of centimeters of rainfall that has fallen since midnight, where $t$ is the time in hours. Interpret the following in practical terms, giving units.

(a) $f(10) = 3.1$

At 10 am, it has fallen 3.1 cm of rainfall.

(b) $f^{-1}(5) = 16$

It has fallen 5 cm of rainfall at 4 pm.

(c) $f'(10) = 0.4$

If we want to look at a second later ($\frac{1}{60} hrs$), the derivate can help to get a good estimation with the following formula

$$f\left(10 + \frac{1}{60}\right) \approx f(10) + \frac{1}{60} 0.4 \text{ cm} = f(10) + 0.0066 \text{ cm}.$$

This means that at 10:01 am, it has fallen approximately 0.0066 cm of rainfall more than it has fallen at 10 am.

(d) $\left(f^{-1}\right)'(5) = 2$

First of all, notice that the units for 5 are cm, and the units for 2 are hr/cm. The statement above gives us then an approximation:

$$f^{-1}(5.1) \approx f^{-1}(5) + 0.1 \cdot 2hr = f^{-1}(5) + 0.2hr.$$

This means that from 5 cm to 5.1 cm of rainfall, it has approximately taken 0.2 hours.

**Problem 4**

(a) Find the average rate of change of $r(x) = \frac{1}{x^2}$ on the interval $-2 \leqslant x \leqslant \pi^2$.

The average rate of change in the interval above is:

$$\frac{r(\pi^2) - r(-2)}{\pi^2 - (-2)} = \frac{\frac{1}{\pi^4} - \frac{1}{4}}{\pi^2 + 2} \approx -0.02$$

(b) Compute *exactly* (do not estimate!) the derivative of $r(x) = \frac{1}{x^2}$ at $x = 3$. Show your work

According to the definition,

$$r'(3) = \lim_{h\to 0} \frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h}.$$

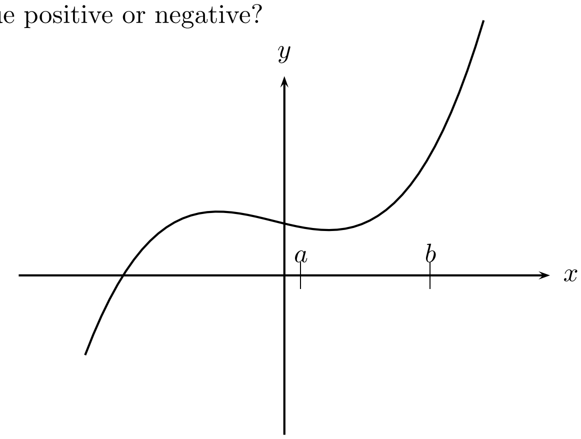
Multiplying both side by $9 \cdot (3+h)^2$, we get

$$r'(3) = \lim_{h\to 0} \frac{9 - (3+h)^2}{h \cdot 0 \cdot (3+h)^2} = \lim_{h\to 0} \frac{(3-3-h)\cdot(3+3+h)}{h \cdot 9 \cdot (3+h)^2} = \lim_{h\to 0} \left(-\frac{6+h}{9\cdot(3+h)^2}\right)$$

$$= -\frac{6}{9\cdot 9} = -\frac{2}{27}.$$

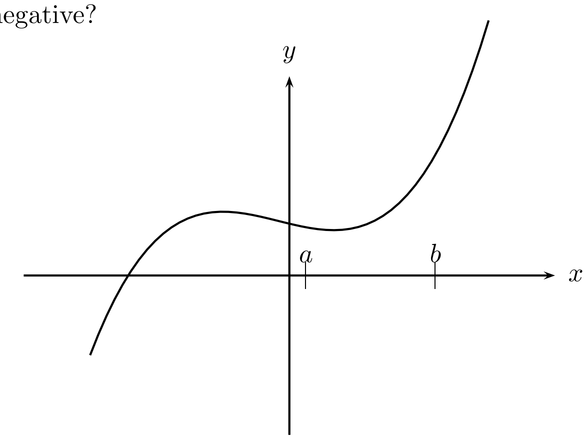
**Problem 5**

(a) Represent the number $\frac{h(b)-h(a)}{b-a}$ on the graph of $h(x)$ below, and indicate *how* it is represented. Be specific! Is this value positive or negative?




The number $\frac{h(b)-h(a)}{b-a}$ is the slope of the line joining the points $(a, f(a))$ and $(b, f(b))$. According to the graph, this number is positive.

(b) Represent the number $h'(a)$ on the graph, and indicate *how* it is represented. Be specific! Is this value positive or negative?




The number $h'(a)$ is the derivative of $h$ at $x = a$, and it represents the slope of the tangent line to the graph of $h$ at the point $(a, h(a))$. According to the graph, this number is negative.