Name:______________________________.

MATH 115 - SEC 011, WINTER 2011. QUIZ 3
TIME LIMIT: 15 MINUTES

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Good luck!

Problem 1

For each problem below, find a value if the constant k such that the limit exists. Show your reasoning.

- $\lim_{x\to 4} \frac{x^2-k^2}{x-4}$

  Since $x-4$ approaches zero as $x \to 4$, then the only chance for the limit to exist is if the limit of the numerator is also zero as $x \to 4$, otherwise the fractional function has a vertical asymptote at $x = 4$. So we need

  $$0 = \lim_{x\to 4} \left(x^2 - k^2\right) = 4^2 - k^2,$$

  So $k = 4$ satisfies the required condition. In fact, if $k = 4$,

  $$\lim_{x\to 4} \frac{x^2-k^2}{x-4} = \lim_{x\to 4} \frac{x^2-4^2}{x-4} = \lim_{x\to 4} \frac{(x-4)\,(x+4)}{x-4} = \lim_{x\to 4}(x+4) = 4+4 = 8$$

- $\lim_{x\to 1} \frac{x^2-kx+4}{x-1}$

  Analogously, we need the limit of the numerator to be zero as $x \to 1$. So, we need

  $$0 = \lim_{x\to 1}(x^2 - k\;x + 4) = 1 - k + 4 = 0$$

  So, $k = 5$ works. In fact, if $k = 5$,

  $$\lim_{x\to 1} \frac{x^2-5x+4}{x-1} = \lim_{x\to 1} \frac{(x-1)\,(x-4)}{x-1} = \lim_{x\to 1}(x-4) = 1-4 = -3$$

- $\lim_{x\to \infty} \frac{x^2+3x+5}{4x+1+x^k}$

  Here, we need to check the long-run behavior of the fractional function. The leading term of the numerator is x^2. If k is a non-negative integer, the denominator is a polynomial. If $k = 0$ or $k = 1$, the order of the polynomial in the numerator would be greater than the order of the polynomial of the denominator, and the limit would be ∞. On the other hand, if $k = 2$, then the leading term of the polynomial in the denominator is x^2. So $k = 2$ works, and in fact

  $$\lim_{x\to \infty} \frac{x^2+3\;x+5}{4\;x+1+x^2} = \lim_{x\to \infty} \frac{x^2}{x^2} = 1$$

Date: January 26, 2011.

Problem 2 In a tine of t in seconds, a particle moves a distance of s meters from its starting point, where $s = 4t^2 + 3$. **Include units.**

(a) Find the average velocity between $t = 1$ and $t = 1 + h$ if

(i) $h = 0.1$

The average velocity is

$$\frac{4(1+h)^2 + 3 - 4 - 3}{h} = \frac{4\ (1+h)^2 - 4}{h}$$
$$\approx 8.4 m/s \text{ if } h = 0.1$$

(ii) $h = 0.01$

The average velocity here is

$$\frac{4\ (1+h)^2 - 4}{h} \approx 8.04 m/s \text{ if } h = 0.01$$

(iii) $h = 0.001$

Finally, the average velocity here is

$$\frac{4\ (1+h)^2 - 4}{h} \approx 8.004 m/s \text{ if } h = 0.001$$

(b) Use your answer to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

As $h \to 0$, the average velocity seems to be converging to

$$8\ m/s$$

Problem 3

(a) Sketch the graph of a continuous function f with all of the following properties:

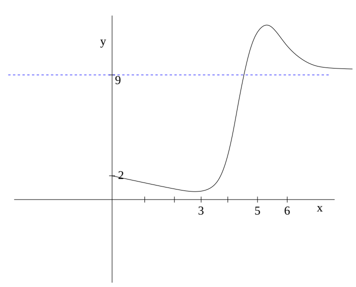
(i) $f(0) = 2$

(ii) $f(x)$ is decreasing for $0 \leq x \leq 3$

(iii) $f(x)$ is increasing for $3 < x \leq 5$

(iv) $f(x)$ is decreasing for $x > 5$

(v) $f(x) \to 9$ as $x \to \infty$



(b) Is it possible that the graph of f is concave down for all $x > 6$? Explain

No. Since the function has to be decreasing for $x \geq 5$, it needs to approach the horizontal asymptote $y = 9$ from above, as can be seen from the graph. As a result, it needs to be concave up for $x >> 0$ big enough. If if was concave down, it would cross the horizontal asymptote and will keep decreasing.