Name:____________________________________.

MATH 115 - SEC 011, WINTER 2011. QUIZ 2
TIME LIMIT: 15 MINUTES

INSTRUCTOR: GERARDO HERNÁNDEZ

Good luck!

Problem 1 One hundred kilograms of radioactive substance decay to 40 kg in 10 years. How much remains after 20 years?

The amount of the radioactive substance (in kilograms) as a function of time is of the form

$$Q = Q_0 a^t,$$

where $Q_0 = 100$ kg is the initial amount, and a is the decay factor, and t is given in years. Since the substance decay to 40 kg in 10 years, then

$$100a^10 = 40.$$

Then $a^10 = 0.4$, which implies

$$a = (0.4)^{\frac{1}{10}}.$$

Therefore

$$Q = 100 \text{ kg } \ (0.4)^{\frac{t}{10}}$$

After 20 years,

$$Q(20) = 100 \text{ kg } \ (0.4)^{\frac{20}{10}} = 16 \text{ kg}.$$

Problem 2 The Bay of Fundy in Canada has the largest tides in the world. The difference between low and high water levels is 15 meters (nearly 50 feet). At a particular point the depth of water, y meters, is given as a function of time, t, in hours since the midnight by

$$y = D + A\ cos(B(t - C))$$

(a) What is the physical meaning of D?

This is the midline, or the average hight between the low and high water level.

(b) What is the value of A?

We know

$$|A| = 7.5\text{meters},$$

So either $A = 7.5$ meters, or $A = -7.5$ meters, depending on the sign of A.

Date: January 21, 2011.

(c) What is the value of B? Assume the time between successive high tides is 12.4 hours.

Since the period is 12.4 hours, then

$$\frac{2\pi}{B} = 12.4,$$

So

$$B = \frac{2\pi}{12.4} \approx 0.506708$$

(d) What is the physical meaning of C?

At time $t = C$, we are in a low or high water level, depending on the sign of A.

Problem 3

(a) If $f(x) = ax^2 + bx + c$, what can you say about the values of a,b, and c if

(1) $(1, 1)$ is on the graph of $f(x)$?

If $(1, 1)$ is on the graph of $f(x)$, this means that substituting $x = 1$ and $y = 1$ the equation above holds, which gives

$$1 = a + b + c,$$

So the condition for a, b, c is simply

$$a + b + c = 1.$$

(2) $(1, 1)$ is the vertex of the graph of $f(x)$? [Hint: The axis of symmetry is $x = -b/(2a)$]

According to the hint, the axis of symmetry in general is $x = -\frac{b}{2a}$. If $(1, 1)$ is the vertex, then $x = 1$ is the axis of symmetry. As a result we get

$$1 = -\frac{b}{2a}, \text{ or } b = -2a$$

Together with the condition above ($a + b + c = 1$) we get

$$\begin{aligned} -a + c &= 1 \\ b &= -2a \end{aligned}$$

(3) The y intercept of the graph is $(0, 6)$?

The $y-$ intercept is simple c. So $c = 6$.

(b) Find a quadratic function satisfying all three conditions.

We need

$$\begin{aligned} -a + c &= 1 \\ b &= -2a \\ c &= 6 \end{aligned}$$

So $c = 6$, $a = c - 1 = 5$, and $b = -2 \cdot 5 = -10$. Therefore

$$y = 5\ x^2 - 10\ x + 6$$