Name:____.

MATH 115 - SEC 011, WINTER 2011. QUIZ 1
TIME LIMIT: 20 MINUTES

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Good luck!

Problem 1 Find an equation for the line through the point $(2, 1)$, which is perpendicular to the line $y = 5x - 3$.

Since the new line is perpendicular, we know $m = -1/5$. So the equation for the line show be

$$y = b - \frac{1}{5}x,$$

and we need to find b. Since the point $(2, 1)$ is on the graph, we know

$$1 = b - \frac{1}{5}(2)$$

So $b = 7/5$, and so

$$y = \frac{7}{5} - \frac{1}{5}x$$

Problem 2 Fin x when $6 \cdot 7^x = 4 \cdot 2^x$. Please give the exact answer, and also express it in decimal form with four significant digits.

$$6 \cdot 7^x = 4 \cdot 2^x \text{ implies } \left(\frac{7}{2}\right)^x = \frac{4}{6} = \frac{2}{3}$$

Taking the natural logarithm on both sides we obtain:

$$ln\left(\frac{2}{3}\right) = ln\left(\left(\frac{7}{2}\right)^x\right) = x\ ln\left(\frac{7}{2}\right),$$

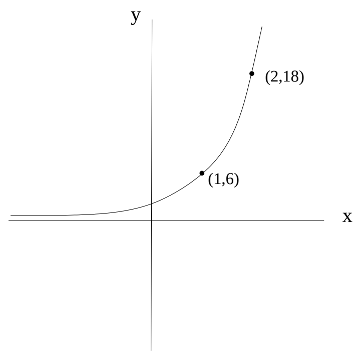
which gives

$$x = \frac{ln\left(\frac{2}{3}\right)}{ln\left(\frac{7}{2}\right)} \approx -0.3237$$

Date: January 14, 2011.

Problem 3 (5 Points)

Find a formula the exponential function that passes through the points $(1, 6)$ and $(2, 18)$.



The exponential function is of the form

$$y = y_0 a^x,$$

where $y + 0$ and a are constants. They need to satisfy

$$y_0 \ a^1 = 6$$
$$y_0 \ a^2 = 18.$$

Diving the two equations gives:

$$\frac{y_0 \ a^2}{y_0 \ a} = \frac{18}{6} = 3$$

So $a = 3$. Also,

$$6 = y_0 \ a^1 = y_0 \ 3. \text{ Then } y_0 = 2.$$

Therefore

$$y = 2 \cdot 3^x$$

Problem 4

A spherical balloon is growing with radius $r = 3t + 1$, in centimeters, for time t is seconds. Find a formula for the volume of the balloon as a function of time (t). Find the volume of the balloon at 3 seconds.

The volume of a balloon of radius r is given by

$$V = \frac{4}{3}r^3.$$

The radius as a function of time is $r = 3\ t + 1$. Composing the two functions, it gives

$$V = \frac{4}{3}\pi\left(3\ t + 1\right)^3$$

Evaluating the function at $t = 3$ seconds, we obtain

$$V(3) = \frac{4}{3}\pi 10^3 \approx 4188.79cm^3$$

Problem 5 A photocopy machine can reduce copies to 80% of their original size. By copying an already reduced copy, further reductions can be made. Estimate the number of times in succession that a page must be copied to make the final copy less than 15% of the size of the original.

The size of the copy after t reductions is given by

$$C(t) = C_0(0.8)^t,$$

where C_0 is the initial size. We want to find an integer t such that

$$(0.80)^t \geq 0.15.$$

Applying the natural logarithm to both sides it gives

$$ln\left((0.8)^t\right) = t\ ln(0.8) \geq ln(0.5)$$

Since $ln(0.8)$ is negative, it gives

$$t \geq \frac{ln(0.15)}{ln(0.8)} \approx 8.50$$

So after $t = 9$ photocopies, the size of the final copy will be les sthan 15%.