**Name:**

# MATH 105 - SEC 001, FALL 2010. QUIZ 6
# TIME LIMIT: 30 MINUTES

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**Good luck!**

**Problem 1.** Give the definition of a even function

An even function is a functions whose is symmetric around the $y$-axis. Algebraically, a function $f(x)$ is even if

$$f(x) = f(-x)$$

for all x

**Problem 2.** Give the definition of an odd function

An odd function is a function whose graph is symmetric through the origin. Algebraically, a function $f(x)$ is odd if

$$f(x) = -f(-x)$$

for all $x$

**Problem 3.** If the graph of $y = e^x$ is reflected about the $y$-axis, what is the formula for the resulting function?

$$y = e^{-x}$$

**Problem 4.** The domain of the function $g(x)$ is $-2 < x < 7$. What is the domain of $g(x-2)$.

$$[0, 9]$$

**Problem 5.** Let $m(n) = n^2 + 3n$. If the graph of $m(n)$ is translated to the right by 3 units, what is the formula for the resulting function? Simplify your answer as much as you can.

$$\begin{aligned} m(n-3) = (n-3)^2 + 3\ (n-3) &= n^2 - 6n + 9 + 3n - 9 \\ &= n^2 - 3n \end{aligned}$$

So the resulting formula is $n^2 - 3n$

**Problem 6.** Express the following in terms of $x$ without natural logs. Give EXACT answers, and simplify them as much as you can.

a) $log(\frac{10}{1000^{5x}})$

$$log(\frac{10}{1000^{5x}}) = log(10) - log(1000^{5x}) = 1 - 5x * log(1000) = 1 - 5x * 3 = 1 - 15x$$

b) $log(\frac{\sqrt{1^{3x}}}{10^{-2x+1}})$

Using the fact that $\sqrt{1^{3x}} = \sqrt{1} = 1$, we get

$$log(\frac{\sqrt{1^{3x}}}{10^{-2x+1}}) = log\left(\frac{1}{10^{-2x+1}}\right) = log(1) - log\left(10^{-2x+1}\right) = 0 - (-2x+1) = 2x - 1$$

c) $e^{xln(10)-x}$

$$e^{xln(10)-x} = e^{xln(10)}e^{-x} = \left(e^{ln(10)}\right)^x e^{-x} = 10^x\ e^{-x}$$

d) $e^{5\ ln(x)-6} + 3log(10^{2x}/100)$

$$e^{5\ ln(x)-6} + 3log(10^{2x}/100) = e^{5ln(x)}e^{-6} + 3[\ log(10^{2x}) - log(100)\ ]$$
$$= \left(e^{ln(x)}\right)^5 e^{-6} + 3[2xlog(10) - 2] = e^{-6}x^5 + 6x - 6$$

**Problem 7.** Find the EXACT answer for the equation:

$$11 \cdot 3^x = 5 \cdot 7^x$$

Applying the natural logarithm to both sides and using its properties, we obtain

$$ln(11) + ln(3^x) = ln(5) + ln(7^x)$$

$$ln(11) + xln(3) = ln(5) + xln(7)$$

Solving this linear equation we get

$$x(ln(3) - ln(7)) = ln(5) - ln(11)$$

and so

$$x = \frac{ln(5) - ln(11)}{ln(2) - ln(7)},\ \text{which is the EXACT solution}$$

**Problem 8.** In 1991, the body of a man was found in melting snow in the Alps of Northern Italy. An examination of a tissue sample revealed that 46 % of the carbon-14 present in his body at the time of his dead had decayed. The half-life of the carbon-14 is approximately 5728 years. How long ago did this man die?

Let's $Q(t) = Q_0\ b^t$ denote the amount of carbon-14 present in his body $t$ years after his dead. First, we need to use the information about the half life to obtain the growth factor $b$. We now that for $t = 5728$, the amount of carbon-14 reduces to half of what it was originally, and so

$$Q(5728) = Q_0 b^{5728} = \frac{1}{2} Q_0,$$

from where we obtain $b^{5728} = \frac{1}{2}$, which implies $b = \left(\frac{1}{2}\right)^{\frac{1}{5728}}$.

Now that we know the growth factor, we can use the rest of the information, which is that when the body was found, 46% of the carbon-14 had decayed, so only 54% was present. So, if we want to know how many years ago this man died, we need to solve

$$Q_0\ b^t = 0.54 * Q_0,$$

or

$$b^t = 0.54.$$

Applying logs to both side we get

$$t\ ln(b) = ln(0.54),$$

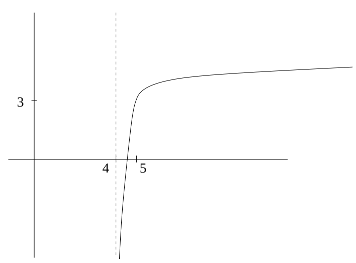
and so

$$t = \frac{ln(0.54)}{ln\left(\left(\frac{1}{2}\right)^{\frac{1}{5728}}\right)} \approx 5092.013 \text{ years}$$

So, this man died approximately in September of 3082 B.C.

**Problem 9.** Graph the following function, and label all asymptotes and intercepts.

$$y = log(x - 4) + 3$$




Vertical asymptote at $x = 4$, no horizontal asymptote, no $y$-intercept and for the $x$-intercept, need to solve $log(x - 4) + 3 = 0$, which implies $log(x - 4) = -3$, so $x - 4 = 10^{-3}$, giving $x = 4.001$.

Problem 10 on next Page

**Problem 10.** Find the hydrogen ion concentration $[H^+]$ for the baking soda used to make donuts that you may be eating now, with a pH of 8.3. Hint: pH=-log$[H^+]$.

Here $pH = 8.3$. So, plugging it into the formula, it gives

$$8.3 = -log[H^+]$$

$$-8.3 = log[H^+],$$

and so $H^+ = 10^{-8.3} \approx 5.01 \times 10^{-9}$