Name:

MATH 105 - SEC 001, FALL 2010. QUIZ 5
TIME LIMIT: 10 MINUTES

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Good luck!

Problem 1

Express the following in terms of x without natural logs. Give EXACT answers, and simplify them as much as you can.

a) $ln(e^{2x})$

Since the exponential is the inverse of the natural log, we have

$$ln(e^{2x}) = 2x.$$

b) $e^{ln(2x+5)+6}$

$$e^{ln(2x+5)+6} = e^{ln(2x+5)}e^6 = e^6\ (2x+5)$$

c) $ln(\frac{1}{e^{5x}})$

$$ln\left(\frac{1}{e^{5x}}\right) = ln\left(e^{-5x}\right) = -5x$$

d) $ln(\frac{\sqrt{e^{3x}}}{e^{-2x+1}})$

$$ln\left(\frac{\sqrt{e^{3x}}}{e^{-2x+1}}\right) = ln\left(e^{3x/2}\right) - ln\left(e^{-2x+1}\right) = \frac{3x}{2} - (-2x+1) = \frac{7x}{2} - 1$$

e) $e^{xln(x)}$

Using the laws of exponents, we get

$$e^{xln(x)} = \left(e^{ln(x)}\right)^x = x^x$$

f) $e^{3\ ln(x)+1} - 2ln(e^{2x}/e)$

Problem 2 in Page 2

$$e^{3\ ln(x)+1} - 2ln(e^{2x}/e) = e^{3ln(x)}e^1 - 2\left(ln(e^{2x}) - ln(e)\right) = e\left(e^{ln(x)}\right)^3 - 2\,(2x-1) = ex^3 - 4x + 2$$

Problem2

A person's blood alcohol content (BAC) is a measure of how much alcohol is in the blood stream. When the person stops drinking, the BAC declines over time as the alcohol is metabolized. If Q is the amount of alcohol and Q_0 is the initial amount, then $Q = Q_o e^{-t/\tau}$, where τ is known as the *elimination time*. How long does it take for a person's BAC to drop from 0.10 to 0.04 if the elimination time is 2.5 hours?

The elimination time is $\tau = 2.5$ hours, so it make sense to express t in hours. We know $Q_0 = 0.1$, and so

$$Q(t) = 0.1e^{-t/2.5}.$$

We want to find t such that

$$Q(t) = 0.1e^{-t/2.5} = 0.04,$$

so applying the natural log to both sides we get

$$ln(0.1) - t/(2.5 \text{ hours}) = ln(0.04),$$

and so

$$t = 2.5 * (ln(0.1) - ln(0.04)) \text{ hours} = 2.5ln(2.5) \text{ hours}$$

An equivalent answer is

$$t = -2.5ln(0.4) \text{ hours}$$