**Name:**

# MATH 105 - SEC 001, FALL 2010. QUIZ 3
# TIME LIMIT: 25 MINUTES

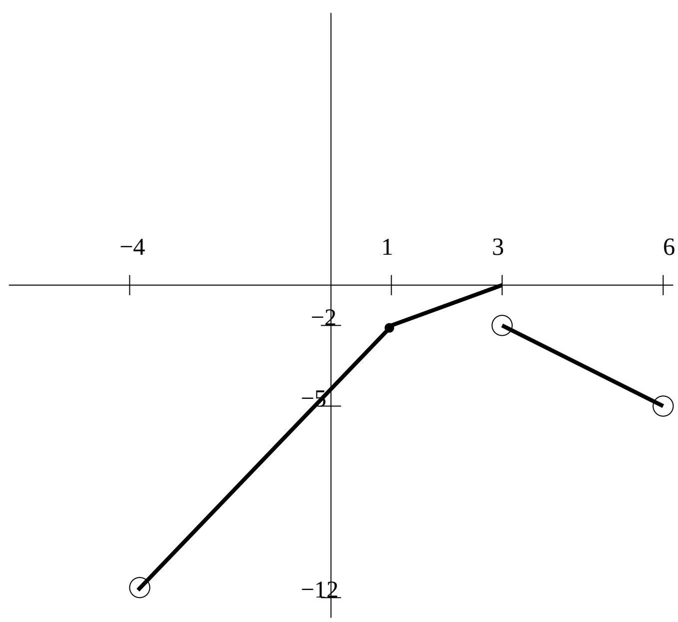
INSTRUCTOR: GERARDO HERNÁNDEZ

**Problem 1**

Consider the following piecewise defined function

$$f(x) = \begin{cases} 2\,x - 4, & -4 < x \leq 1 \\ x - 3, & 1 < x \leq 3 \\ 1 - x, & 3 < x < 6 \end{cases}$$

(a)(3 pts) Sketch the graph of this function




(b)(3 pts) Find the domain and range of this function

The domain is $(-4, 6)$ and the range is $(-12, 0]$

*Date*: September 28, 2010.

**Problem 2 (14 points)**

In order to gain popularity among students, a brand new on-campus hair salon plans to offer a special promotion. The cost of a haircut, in dollars, at the salon as a function of time, in days since February 10th may be described as

$$C(t) = \begin{cases} 9, \ 0 \leq t \leq 3 \\ \\ 9+t, \ 3 < t \leq 8 \\ \\ 20, \ 8 < t < 28 \end{cases}$$

(Assue $t$ takes whole numbers values.)

(a) (3 pts.) If you want them to give them a try, on what date(s) should you have a haircut in order to get the best price?

As we can see from the formula, the best price is 9 dollars, and this happens in February 10th trough February 13th.

(b) (2 pts.) How much will a haircut cost on Feb. 18th?

17 dollars

(c) (2 pts.) On what date will a haircut cost 13 dollars?

February 14

(d) (3 pts.) The cost of a haircut at least $A$ dollars $B$ days into the promotion. Write an expression that describes this sentence using function notation and mathematics symbols only.

$C(B)$ is the cost of the haircut $B$ days into the promotion, and it is at least $A$ dollars. That means that it could be $A$ dollars or something higuer. In symbols, it can be expressed as

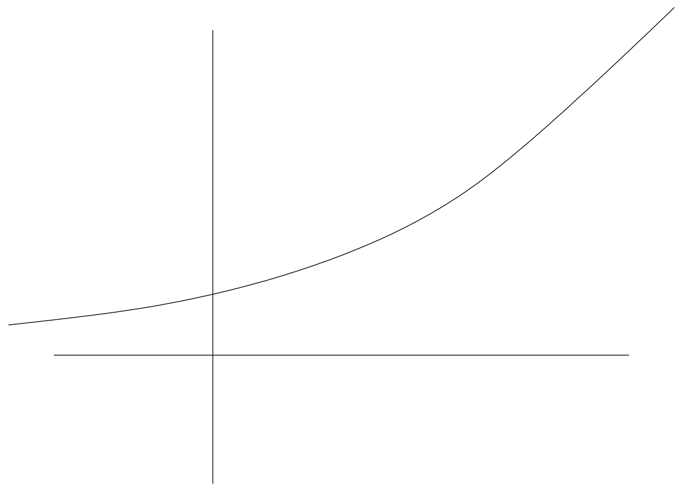
$$C(B) \geq A$$

(e) (4 pts) Calculate $C(9) - C(8)$ and interpret its meaning in the context of the problem.

8 falls into the second category, and 9 into the third one. So $C(9) - C(8) = \$20 - \$17 = \$3$. In words, it means that the difference in price of a haircut between February 18 and February 19 is \$3.

**Problem 3**

(3 pts).Sketch a graph which is everywhere positive, increasing, and concave up.



**Problem 4.**

(4 pts.) Let $P = f(t)$ be the population in millions in year $t$. Assume this function is invertible. Give the **meaning** and **units** of the inverse function.

Symbolically, it can be written as $f = f^{-1}(P)$ where $t$ is in years and $P$ is in millions, and is the size of the population. The meaning here is that this function tells you the year $t$ when the population is $P$ millions (*in this order*).

**Problem 5.**

(4 Pts). Find the zeros of $Q(x) = -5x + 2x^2 - 3$ using the quadratic formula.

This equation can also be written as

$$2x^2 - 5x - 3 = 0,$$

and we can find the zeros using the quadratic formula:

$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{5 \pm \sqrt{49}}{4}$$

and so $x = 3$ or $x = -1/2$.

**Problem 6**

(4 Pts). Determine the concavity of the graph of $f(x) = 4 - x^2$ between $x = -1$ and $x = 5$ by calculating average rates of change over intervals of length 2.

Intervals of length 2 are $[-1, 1], [1, 3], [3, 5]$ and the rate of change in those intervals are

$$\frac{f(1) - f(-1)}{2} = 0, \ \frac{f(3) - f(1)}{2} = -4, \ \frac{f(5) - f(3)}{2} = -8.$$

Since the rate of change in these intervals is decreasing, the function is concave down.