# MATH 105 - SEC 001, FALL 2010. QUIZ 1

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## Problem 1 (5 points)

Thomas Gross is a researcher in the Department of Cellular, Molecular and Developmental Biology here at Michigan; you may soon also know him as the guy playing the harmonica and washboard outside the UGLi (the Undergraduate Library).

A few years back, the Michigan Daily did some investigative reporting and discovered the following facts: The amount of time $G(d)$, in minutes, that Mr. Gross plays is a linear function of $d$ (here $d$ refers to Fahrenheit degrees). Reporters for the daily observed that Mr. Gross played for two hours and 15 minutes when the average daily temperature was 92° F and that he played for one hour when the average daily temperature was 32° F.

(1) Find a formula for $G(d)$ as a function of $d$ when $t \geq 0$.

The slope is given by $m = \frac{(135-60)min}{92^\circ - 32^\circ} = 1.25\ min/\ ^\circ F$. The y-intercept can be found by evaluating the linear function at $d = 32^\circ F$.

$$60min = b + (1.25\ min/\ ^\circ F)32^\circ F = b + 40min \longrightarrow b = 20min$$

Therefore

$$G(d) = 20min + (1.25\ min/\ ^\circ F)\ d$$

(2) Calculate and interpret the slope of the graph of $G(d)$. Include units.

$m = 1.25\ min/\ ^\circ F$ means that Mr. Gross plays 1.25 extra minutes every time the daily average temperature increases by $1^\circ F$.

(3) Calculate and interpret $G(0)$. Include units.

$G(0) = 20\ min$ means that Mr. Gross plays 20 minutes when the average temperature is $0^\circ F$.

*Date*: September 15, 2010.

(4) What is the average daily temperature on a day when Mr. Gross plays for 2 hours? Include units.

We need to solve the equation

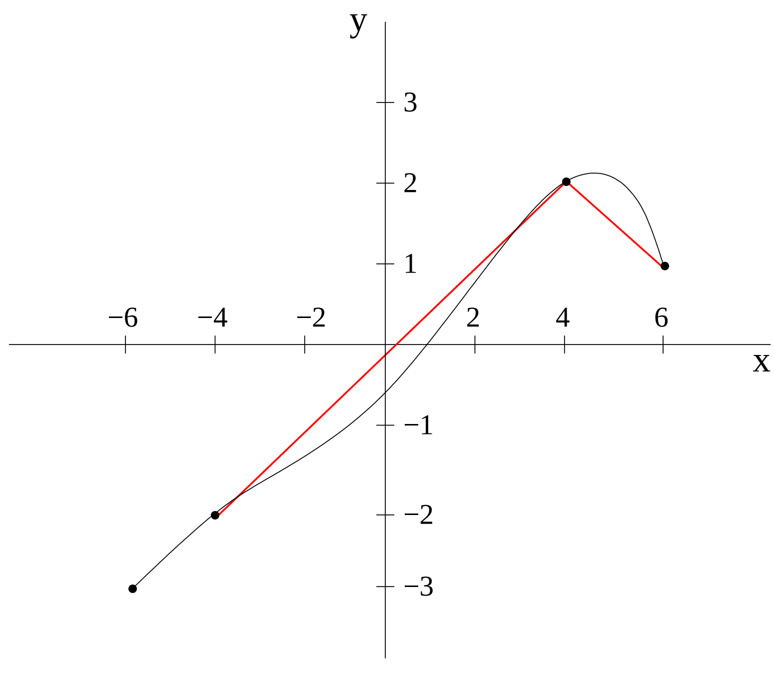
$$120min = 20min + (1.25\ min/\ {}^{\circ}F)d,$$

which solution is

$$d = \frac{120min - 20min}{1.25\ min/\ {}^{\circ}F} = 80^{\circ}F$$

## Problem 2 (4 Points)

The figure below shows the graph of the function $g(x)$.




(1) Estimate $\frac{g(6)-g(4)}{6-4}$

$\frac{g(6)-g(4)}{6-4} = \frac{1-2}{6-4} = \frac{-1}{2}$

(2) The ratio in part (1) is the slope of a line segment joining two points in the graph. Sketch this line segment on the graph.

See graph

(3) Estimate the rate of change for this function over the interval $[-4, 4]$ ($a = -4$ and $b = 4$).

The rate of change is given by

$$\frac{g(4) - g(-4)}{4 - (-4)} = \frac{2 - (-2)}{4 - (-4)} = \frac{4}{8} = 1/2$$

(4) On the graph, sketch the line segment whose slope is given by the ratio in part (c).

See the graph

## Problem 3 (5 Points)

For the following statements, decide whether they are true or false. If the statement is true, give a reason why. If it is false, provide an example where it is not true.

(1) A function must be defined by a formula.

False. Rule of four

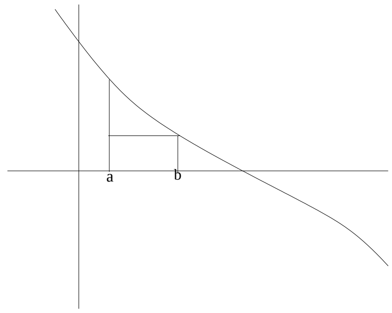
(2) If $f$ is a decreasing function, then the average rate of change of $f$ on any interval is negative.

True. We can see it clearly in the figure below (Figure 2).

(3) The average rate of change of $f(x) = 10 - x^2$ between $x = 1$ and $x = 2$ is the ratio $\frac{10-2^2-10-1^2}{2-1}$.

False. It is

$$\frac{10 - 2^2 - 10 + 1^2}{2 - 1}$$




(4) The following table demonstrates the relationship between two quantities $P$ and $Q$

| $P$ | 0 | 1 | 2 | 3 | 5 |
|---|---|---|---|---|---|
| $Q$ | 5 | 12 | 0 | 12 | 1 |

This table shows that $P$ is a function of $Q$ and that $Q$ is a function of $P$.

False. The vertical line test says $Q$ is a function of $P$, but $P$ is not a function of $Q$.

## Problem 4 (2 Points)

You are looking at the graph of $y$, a function of $x$.

(1) What is the maximum number of times that the graph can intersect the $y$-axis? Explain.

One, because of the vertical line test

(2) Can the graph intersect the $x$-axis an infinite number of times? Explain

Yes. The constant function $f(x) = 0$ is an example